Neoclassical Growth, Environment and Technological Change: The Environmental Kuznets Curve*

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Abstract

The paper investigates socially optimal patterns of economic growth and environmental quality in a neoclassical growth model with endogenous technological progress. In the model, the environmental quality affects positively not only to utility but also to production. However, cleaner technologies can be used in the economy whether a part of the output is used in environmentally oriented R&D. In this framework, if the initial level of capital is low then the shadow price of a cleaner technology is low relative to the cost of developing it given by the marginal utility of consumption and it is not worth investing in R&D. Thus, there will be a first stage of growth based only on the accumulation of capital with a decreasing environmental quality until the moment that pollution is great enough to make profitable the investment in R&D. After this turning point, if the new technologies are efficient enough, the economy can evolve along a balanced growth path with an increasing environmental quality. The result is that the optimal investment pattern supports an environmental Kuznets curve.

Keywords: neoclassical growth model, endogenous technological progress, external effects, environmental Kuznets curve.

JEL Classification System: O33, O41, Q55, Q56.

1 Introduction

Since the publication in the early nineties of several works suggesting that some pollutants follow an inverse-U-shaped pattern relative to countries' incomes, see, for instance, World Bank (1992), Selden and Song (1994) and Grossman and Krueger (1995), the issue of the pollution-income relation has become one of the most important subjects in the field of environmental economics. Although the literature on the issue is mainly empirical, several papers have offered a theoretical explanation of this phenomenon, see John and Pecchenino (1994), Selden and Song (1995) and Stokey (1998).[1] In the first two papers, non-negativity constraints apply on expenditure on pollution abatement for a given technology. John and Pecchenino (1994) study the effects of non-negativity constraints on the evolution of environmental quality in the framework of an overlapping generations model where pollution depends positively on consumption and negatively on abatement, whereas Selden and Song (1995) use the neoclassical growth model with pollution first analyzed by Forster (1973) where pollution depends positively on capital and negatively on abatement.[2] However, in Stokey's (1998) paper an AK model with technological change is analyzed. She assumes that there is a continuous of technologies that can be used. The different technologies are characterized by an index, between zero and unity, that defines the emission rate for the production process. In this framework an EKC appears because during a first stage it is optimal to produce with the dirtiest technology. Nevertheless, although sustained growth is possible it is not optimal. The explanation for why growth ceases is related to the fact that after substituting the pollution function into the production function, this presents constant returns to scale with respect to the capital stock and the pollution but a decreasing marginal productivity for capital.

In this paper, we extend the neoclassical growth model proposed by Forster (1973)

[1] A theoretical explanation of the EKC can be already found in Gruver (1976). This author analyzes a linear growth model with two types of capital, the productive capital and the pollution control capital. However, as the pollution control capital is not productive, the economy evolves to an steady state without growth.

[2] More recently, Lieb (2004) has extended John and Pecchenino's (1994) model to two pollutants, a stock pollutant as in John and Pecchenino (1994) and a flow pollutant.

in two directions. First, we assume that environmental quality affects not only to consumer's welfare but also to production through its positive effect on the productivity of labor and also on the productivity of capital. This positive effect of the environment on productivity was already considered in the World Bank (1992) report. In this report it is recognized that the main channel through which pollution affect productivity is through its negative effect on human health. Poor health causes considerable economic losses not only because it affects workers' participation in the labour market and their productivity, but also because it affects worker's learning abilities. On the other hand, a lower quality of the environment can increase physical depreciation or cause important losses of the capital stock as a consequence of the extreme meteorological phenomena. In accordance with this perspective we assume that environmental quality is (indirectly) a production factor. Mohtadi (1996) and Smulders and Gradus (1996) follow this approach in their analysis of pollution abatement on growth.[3] More recently Cassou and Hamilton (2004) have also adopted this approach in their investigation of the optimal patterns of economic development in a two-sector endogenous growth model with clean and dirty goods. A second extension is that we assume that cleaner technologies can be used in the economy whether a part of the output is used in environmentally oriented R&D. In this case, as environmental quality affects production, the investment in R&D to develop cleaner technologies also enhances the productivity of capital and labor. For this growth model with endogenous technological progress, we find that, for a given initial technology and capital stock, if the initial level of capital is low then the shadow price of a cleaner technology is low relative to the cost of developing it and it is not worth investing in R&D. Thus, there will be a first stage of growth based only on the accumulation of physical capital with an increasing pollution until the moment that pollution is great enough to make profitable the investment in R&D to develop cleaner technologies. Then if the new technologies are efficient enough it is possible to growth along a balanced growth path (BGP) with an increasing environmental quality. The result is that the optimal investment pattern supports an inverted-U-shaped pattern of pollution to countries' incomes.

[3]Schou (2000) also follows this approach in his study of the effects of pollution on long-run growth when pollution is caused by the use of a non-renewable resource.

In the paper we also investigate which can be the effects of greener preferences on growth. Our investigation shows that the effect of greener preferences on growth can be positive or negative depends on the degree of environmental conscience of consumers. In particular, we find that the lower the degree of environmental conscience of consumers, the greater the possibilities that greener preferences lead to an increase in the growth rate. Nevertheless, for the numerical simulation developed in Section 4 we find that there is no conflict between the environmental preservation and the economic growth whatever is the degree of environmental conscience of consumers. The positive effect of greener preferences on growth occurs because greener preferences increase the investment in R&D causing an increase in productivity that can support a greater rate of growth for the economy. Another characteristic of our model is that it presents multiple long-run equilibria (global indeterminacy). We show that this characteristic arises because of the positive externality of environmental quality on consumer's welfare. Nevertheless, we find that only for one of the two equilibria the model presents, a sustained growth is guaranteed. Finally, we would add that along the BGP our model behaves as an AK model since the output can be written as a linear function of capital. This property explains why growth in our model is compatible with an increasing environmental quality whereas this is not the case in Stokey's (1998) model that in the long-run behaves as a neoclassical growth model with decreasing marginal productivity for capital. As a consequence also of this property, we find that the BGP is unstable.

Several papers have studied the relationship among economic growth, environment and technological change. See, for instance, Bovenberg and Smulders (1995,1996), Grimaud (1999), Reis (2001), Hart (2004), and more recently Ricci (2007) and Cunha-e-sá and Reis (2007), Grimaud and Rouge (2008) and Reis et al. (2008). However, any of these papers look for an explanation of the EKC. Nevertheless, our paper is close of that written by Reis (2001). Although the model and the aim of the investigation developed by Reis is different to the model and the aim addressed in this paper, we look at the investment in R&D in the same way except that we do not assume that the elasticities of pollution function are equal to unity as it is assumed in Reis' (2001) paper.

Other papers that include technological change in the theoretical analysis of the EKC are Jones and Manuelli (2001), Cassou and Hamilton (2004) and Hartman and Kwon (2005).[4] However, Jones and Manuelli (2001) focus, in the framework of an overlapping generations model with a continuum of technologies indexed by "cleanliness", on showing how different decision-making institutions can affect the pollution-income relationship. They find that voting over (proportional) effluent charges can generate an EKC. Cassou and Hamilton (2004) investigate privately and socially optimal patterns of economic growth in a two-sector endogenous growth model with clean and dirty goods. They consider a second-best fiscal policy framework in which distortionary taxes jointly influence economic growth and environmental quality. In this policy setting they obtain the conditions that produce an EKC. In their model these conditions do not arise with a consumption externality as it occurs in our model. Hartman and Kwon (2005) adopt Stokey's (1998) approach in the framework of a growth model with physical and human capital where the human capital accumulation does not depend on pollution. Consequently, the result is that an optimal sustained growth can be supported by the accumulation of human capital.

Finally, we would like to present some comments on the scope of the paper and the implications for the environmental policy that can be derived from our results. A first thing we would like to clarify is that our model could yield different results for the relationship between pollution and income depending mainly on the initial conditions and on the cost of pollution abatement. Thus, the EKC is one of the patterns that pollution can adopt. Other patterns could be optimal, but what we want to highlight in this paper is that beginning from a pristine natural environment, it can be optimal to postpone investment in abatement technology until a critical level of pollution is reached so that, once this level is reached, if the costs of pollution abatement are not very high, the economy could evolve along a BGP with increasing environmental quality. A second point we would like to comment is that we cannot conclude from our model that just

[4] Kelly (2003) shows numerically how the shape of the emissions and pollution stock curves varies with pollution specific parameters in the framework of a neoclassical growth model with constant population and technology. For some parameter values he obtains an inverted U-shaped income-environment relation when the measure of pollution is emissions.

leaving the economy to growth the environmental problems as the climate change will be solved in the future. The reasom why growth cannot be the solution for climate change is that the environmental quality is a public good and as it is well known then the decentralized equilibrium is not optimal. With external effects in production and consumption, polluting firms do not select the optimal level of investment in abatement technology so that even if the conditions for the appearance of an EKC are satisfied the decentralized equilibrium could yield a different pattern of pollution to income with increasing pollution. According to our results what we need to face the climate change challenge is a well-defined environmental policy that promotes investment in abatement technologies.

The paper is organized as follows. In the next Section the model is presented. In Section 3, the interior central planner solution is derived and the existence, multiplicity and stability of the long-run equilibria are studied. The section ends with an investigation of the effects on growth of greener prferences. In Section 4, the transitional dynamics of the model is analyzed and the conditions to obtain an EKC are established. This Section also includes a numerical illustration with a sensitivity analysis of the results. Finally, Section 5 contains the conclusions and future lines for research are mentioned.

2 The model

We consider a closed economy with constant population normalized to one. The intertemporal utility of the representative consumer is given by

$$\int_0^\infty U(Q,C)e^{-\rho t}dt, \tag{1}$$

where Q stands for the flow of environmental services, C for per capita consumption, and the parameter $\rho > 0$ for the rate of time preference.[5] For simplicity, we assume that $U(Q,C)$ is additively separable and logarithmic,

$$U(Q,C) = \phi \ln Q + \ln C, \;\; \phi > 0, \tag{2}$$

[5]The time argument has been suppressed in this an all subsequent equations if no ambiguity arises.

so that for a given combination (Q, C) the greater ϕ the greater the marginal rate of substitution of consumption for environmental services (MRS_{CQ}) and the consumer cares more about the environment. For this utility function preferences are homothetic and indifference curves strictly convex. Consequently, MRS_{CQ} is decreasing.[6] Moreover, the intertemporal elasticity of substitution is equal to unity. See e.g. Cassou and Hamilton (2004), Grimaud and Rouge (2008) and Reis et al. (2008) for a logarithmic utility specification.

The production function is given by

$$Y = G(Q, K) = AQ^{\chi}K^{\beta}, \quad A > 0,\ \chi > 0 \text{ and } \beta \in (0, 1), \tag{3}$$

where K is the capital-labor ratio. The parameter χ represents the positive effect of environmental quality on production.[7] According to this specification we are assuming that natural environment is (indirectly) a factor of production so that the higher the pollution, the lower the environmental quality provided by Q and the lower the output for the same amount of capital and labor. Pollution can affect either the productivity of labor or/and deteriorate the physical capital hence for the same amounts of factors, the output would be inversely related with pollution.

Following Forster's model, the environmental quality depends negatively on capital, to capture the polluting consequences of economic activity,

$$Q = Q(K, z) = K^{-\lambda}z^{\gamma},\ \lambda, \gamma > 0, \tag{4}$$

where z is an index of the abatement technology used in the economy and λ, γ stand for the elasticities of Q with respect to K and z.[8] An increase in z implies an increase in environmental services for the same stock of capital so that a higher z means a cleaner

[6] Some authors as Gradus and Smulders (1996) have incorporated a lower bound in environmental quality that must be satisfied to sustain normal life and production. This is a natural assumption in growth models with pollution. However, it is not so relevant when the long-run equilibrium supports an inverted U-shaped pattern of pollution to income as it occurs for the growth model solved in this paper.

[7] In Forster (1973) and Selden and Song (1995) it is assumed that the environmental quality has no effect of production.

[8] The specification of functions (3) and (4) were used by Smulders and Gradus (1996) in their analysis

technology. But as environmental quality also affects the productivity of capital, a higher z also means a higher output for the same stock of capital. This positive effect of technological progress on production can be explicitly recognized by substituting (4) in (3) that yields: $Y = Az^{\gamma\chi}K^{\beta-\lambda\chi}$ where we assume that $\beta > \lambda\chi$ in order to guarantee that the marginal productivity of capital is positive. According to this production function the rate of growth of the output is given by

$$g_Y = (\beta - \lambda\chi)g_K + \gamma\chi g_z,$$

where g_z is the rate of technological progress. In this case, the economy only could reach a BGP with $g_Y = g_K = g_z$ provided that $\beta - \lambda\chi + \gamma\chi = 1$. From now, we assume that this condition holds and we focus on the investigation of whether there exists a BGP and which is the transitional dynamics. As $\beta < 1$, this condition requires that $\lambda < \gamma$ that according to (4) implies that environmental quality is increasing along the BGP. In other words, if the marginal productivity of capital is decreasing, it is only possible to reach a BGP in the neoclassical framework if the elasticity of Q with respect to z is greater than the elasticity of Q with respect to K.

Under this assumption, the production function can be written as

$$Y = F(z, K) = Az^{1-\alpha}K^{\alpha}, \quad 1 - \alpha = \gamma\chi, \; \alpha = \beta - \lambda\chi, \quad \alpha \in (0, 1) \tag{5}$$

that it could be also written in terms of per unit of effective labor as $y = Ak^{\alpha}$ where $y = Y/z$ and $k = K/z$. Notice that this specification for the production function coincides with the neoclassical production function with labor-augmenting technological progress with the particularity that in our model the rate of technological progress is endogenous.

We now consider that the development of cleaner technologies is the result of investment in R&D. We consider that part of the output is used in a R&D sector and determine the optimal rate of technological progress. The equation of motion for capital is:

$$\dot{K} = Az^{1-\alpha}K^{\alpha} - C - I_z, \quad K(0) = K_0 > 0. \tag{6}$$

of the effects of pollution control on long-run growth, although for these authors z stands for a control variable (pollution abatement) whereas in this paper z stands for a state variable (abatement technology). This approach corresponds to the one adopted by Reis (2001).

where I_z is the investment in R&D.[9] Thus, the rate of continuous technological progress is endogenously determined through the decisions of investment in R&D:

$$\dot{z} = I_z, \quad z(0) = z_0 > 0. \tag{7}$$

Reis (2001) assumes that investment in R&D is irreversible. This is a natural assumption in our model as well as the economy can only grow in the long run if the investment in R&D is positive. Nevertheless, we show that this assumption is not crucial for the results obtained in this paper since along the BGP is not optimal to revert to dirtier technologies. In fact, the only constraint we need to obtain an EKC is the irreversibility of the initial value of z. To assume that z_0, that can be interpreted as the dirtiest technology available, is a lower bound for z is enough to obtain that it could be optimal not to invest in technological progress during a first stage.

3 Endogenous technological progress

We now derive the central planner solution. This solution maximizes the utility of the representative consumer (1), subject to (6) that describes the dynamics of capital and (7) that shows the evolution of z, given the initial conditions.

Let H be the current-value Hamiltonian of the central planner's problem

$$H = U(Q(K,z),C) + \nu(F(z,K) - C - I_z) + \mu I_z,$$

where ν and μ are the shadow prices of K and z. The first-order necessary conditions are[10]

[9] The assumption of zero depreciation has no qualitative effects.

[10] For this problem the necessary conditions are not sufficient since the concavity of the Hamiltonian is not guarantee for all non-negative values of (K, z, C, I_z). Notice that although the utility function is concave, function (4) is convex with respect to K so that after eliminating Q from the utility function the resulting function $U(Q(K,z),C)$ is convex with respect to K.

$$
\begin{aligned}
\frac{\partial H}{\partial C} &= U_C - \nu = 0, && (8)\\
\frac{\partial H}{\partial I_z} &= -\nu + \mu \leq 0,\ I_z \geq 0,\ (\mu - \nu) I_z = 0, && (9)\\
\dot{\nu} &= \rho\nu - (U_Q Q_K + \nu F_K), && (10)\\
\dot{\mu} &= \rho\mu - (U_Q Q_z + \nu F_z), && (11)
\end{aligned}
$$

plus the transversality conditions

$$
\lim_{t \to +\infty} \nu K e^{-\rho t} = \lim_{t \to +\infty} \mu z e^{-\rho t} = 0. \tag{12}
$$

Condition (9) implies that if $\nu > \mu$ then $I_z = 0$, meaning that if the shadow price of a cleaner technology is too low relative to the cost of developing it given by the marginal utility of consumption, then this technological development is not worth investing in. Thus, for developing cleaner technologies it is necessary that $\nu = \mu$. In this case it follows from (10)-(11) that the net returns on investment in capital and in R&D must be equal

$$
U_Q Q_K + U_C F_K = U_Q Q_Z + U_C F_z,
$$

where the net return on investment in capital is given by the positive value of the marginal productivity of capital, $U_C F_K$, less the value of the negative effect that the capital has on utility through the deterioration of environmental quality, $U_Q Q_K$, whereas the net return on investment in R&D is given by the positive value of the marginal productivity of a cleaner technology, $U_C F_z$, plus the value of the positive effect that a cleaner technology has on utility through an improvement of environmental quality, $U_Q Q_z$.

For the functions of the model this condition yields:

$$
-\frac{\phi\lambda}{K} + \frac{1}{C}\alpha A \left(\frac{K}{z}\right)^{\alpha-1} = \frac{\phi\gamma}{z} + \frac{1}{C}(1-\alpha) A \left(\frac{K}{z}\right)^{\alpha}, \tag{13}
$$

that can be rewritten as

$$
-\phi\lambda x + \alpha A k^{\alpha-1} = \phi\gamma k x + (1-\alpha) A k^{\alpha}, \tag{14}
$$

where x is the ratio of consumption to capital and k the capital per unit of effective labor.

Condition (13) defines implicitly an optimal policy function for x :

$$x(k) = \frac{(\alpha - (1-\alpha)k)A}{\phi(\lambda k^{1-\alpha} + \gamma k^{2-\alpha})}, \tag{15}$$

This rule establishes that to obtain a positive ratio of consumption to physical capital, k must be lower than $\alpha/(1-\alpha)$.

Differentiating (15) with respect to time, the rate of growth of x is obtained as a function of the rate of growth of k,

$$\frac{\dot{x}}{x} = -\frac{p_1(k)}{p_2(k)}\frac{\dot{k}}{k}, \tag{16}$$

where

$$\begin{aligned} p_1(k) &= \gamma(1-\alpha)^2k^2 - \alpha((1-\alpha)\lambda + (2-\alpha)\gamma)k - \alpha(1-\alpha)\lambda, && (17)\\ p_2(k) &= \gamma(1-\alpha)k^2 + ((1-\alpha)\lambda - \alpha\gamma)k - \alpha\lambda. && (18) \end{aligned}$$

From (8) and (10) the rate of growth of consumption is calculated that using (15) can be written as

$$\frac{\dot{C}}{C} = \frac{(\alpha\gamma + (1-\alpha)\lambda)Ak^{\alpha}}{\lambda + \gamma k} - \rho, \tag{19}$$

so that the rate of growth of x can be also expressed as

$$\frac{\dot{x}}{x} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \frac{(\alpha\gamma + (1-\alpha)\lambda)Ak^{\alpha}}{\lambda + \gamma k} - (\rho + g_K), \tag{20}$$

where $g_K = \dot{K}/K$.

Then equaling (20) to (16), the rate of growth of k can be written as a function of the rate of growth of capital.

$$\frac{\dot{k}}{k} = -\left(\frac{(\alpha\gamma + (1-\alpha)\lambda)Ak^{\alpha}}{\lambda + \gamma k} - (\rho + g_K)\right)\frac{p_2(k)}{p_1(k)}. \tag{21}$$

On the other hand, using the resource constraint and the optimal policy function for x, the rate of technological progress, g_z, can be obtained as a function of the rate of growth of the capital

$$g_z = \frac{(\phi\gamma + 1 - \alpha)Ak^{1+\alpha} - (\alpha - \phi\lambda)Ak^{\alpha}}{\phi(\lambda + \gamma k)} - kg_K, \tag{22}$$

so that the rate of growth of k can be also written as

$$\frac{\dot{k}}{k} = g_K - g_z = (1+k)g_K - \frac{(\phi\gamma + 1 - \alpha)Ak^{1+\alpha} - (\alpha - \phi\lambda)Ak^{\alpha}}{\phi(\lambda + \gamma k)}. \tag{23}$$

Then equaling (23) to (21) the optimal policy function for g_K is obtained

$$\begin{aligned} g_K(k) \;=\;& \frac{(\phi\gamma + 1 - \alpha)Ak^{1+\alpha} - (\alpha - \phi\lambda)Ak^{\alpha}}{\phi(\lambda + \gamma k)((1+k)p_1(k) - p_2(k))} p_1(k) \\ &+ \frac{\phi\rho(\lambda + \gamma k) - \phi(\alpha\gamma + (1-\alpha)\lambda)Ak^{\alpha}}{\phi(\lambda + \gamma k)((1+k)p_1(k) - p_2(k))} p_2(k). \end{aligned} \tag{24}$$

Finally, (24) can be used in (22) to obtain the optimal policy function for g_z :

$$\begin{aligned} g_z(k) \;=\;& \frac{(\phi\gamma + 1 - \alpha)Ak^{1+\alpha} - (\alpha - \phi\lambda)Ak^{\alpha}}{\phi(\lambda + \gamma k)((1+k)p_1(k) - p_2(k))} p_1(k) \\ &- \frac{\phi\rho(\lambda k + \gamma k^2) + (1-\alpha)(1 + \phi(\gamma - \lambda))Ak^{1+\alpha} - (\alpha - \phi\lambda)Ak^{\alpha}}{\phi(\lambda + \gamma k)((1+k)p_1(k) - p_2(k))} p_2(k) \end{aligned} \tag{25}$$

and by difference, $g_K(k) - g_z(k)$, the rate of growth of k :

$$\begin{aligned} \frac{\dot{k}}{k} \;=\;& \frac{\phi\rho(\lambda + (\lambda + \gamma)k + \gamma k^2)}{\phi(\lambda + \gamma k)((1+k)p_1(k) - p_2(k))} p_2(k) \\ &- \frac{(1 + \phi(\gamma - \lambda))(\alpha - (1-\alpha)k)Ak^{\alpha}}{\phi(\lambda + \gamma k)((1+k)p_1(k) - p_2(k))} p_2(k), \end{aligned} \tag{26}$$

This result establishes that the dynamics of the model can be summarized in the first-order differential equation (26) and initial condition $k_0 = K_0/z_0$.

3.1 Long-run equilibrium and global indeterminacy

This subsection focuses on a long-run general equilibrium in which the economy can growth a constant rate. An interior BGP is a temporal path defined by a vector $(\hat{k},\ \hat{x},\ \hat{g})$ such that $\hat{k}$ solves $\dot{k} = 0$, the transversality conditions are satisfied, and the rate of growth of environmental quality $\hat{g}_Q$ is given by $(\gamma - \lambda)\hat{g}$ according to (4).

The stationary condition, $\dot{k} = 0$, yields the following equation[11]

$$\phi\rho(\lambda + (\lambda + \gamma)\hat{k} + \gamma\hat{k}^2) = (1 + \phi(\gamma - \lambda))(\alpha - (1-\alpha)\hat{k})A\hat{k}^{\alpha}. \tag{27}$$

[11] We do not take into account the solution given by equation $p_2(\hat{w}) = 0$ since according to (16) the rate of growth of the ratio of consumption to capital is not well defined in this case. Notice that, according to (16), for $p_2(w) = 0,\ \dot{x}/x$ is equal to $-\infty \cdot 0$.

The main task is to solve this equation for $\hat{k}$. The study of this equation leads to the following results[12]

Proposition 1 *For any vector* $(\alpha, \gamma, \lambda, \rho, \phi)$*, a positive lower bound* $\tilde{A}$ *can be defined such that: (i) if* $A > \tilde{A}$ *then there exist two solutions,* $\hat{k}_1$ *and* $\hat{k}_2$*, where* $0 < \hat{k}_1 < \hat{k}_2 < \alpha/(1-\alpha)$*; (ii) if* $A = \tilde{A}$ *then there exists a unique solution,* $\hat{k}$*,where* $0 < \hat{k} < \alpha^2/(1-\alpha^2)$.

$\Rightarrow$ *FIGURE* 1 $\Leftarrow$

Case (i) is shown in Fig. 1. Therefore, we can have multiple (two) long-run values for the capital per unit of effective labor. In turn, each value supports a different BGP, at which consumption and capital grow at a common constant rate of technological progress. Thus, we have

Corollary 1 *Under the conditions in Proposition 1, case (i), there exist multiple BGPs and hence there is global indeterminacy.*

To understand what causes multiplicity, it is convenient to consider two particular cases of this model. First, when the consumer does not care about the environment, i.e. when $\phi = 0$ in (2) above, optimality condition (13) implies that $\hat{k}$ is unique and equal to $\alpha/(1-\alpha)$ in all time periods. According to Prop. 1, this means that the long-run value of k for this particular case is an upper bound for the solutions of our model. Without a negative effect of pollution on consumer's utility, investment in R&D will be lower in relative terms yielding a greater $\hat{k}$. For the second particular case, environmental quality does not affect production, i.e. $\chi = 0$ in (3). In this case, there exist also multiple BGPs and consequently global indeterminacy. Notice that according to (5) $\alpha = \beta$ and $1-\alpha = 0$ if $\chi = 0$. In this case Eq. (27) is written as

$$\phi\rho(\lambda + (\lambda+\gamma)\hat{k} + \gamma\hat{k}^2) = (1 + \phi(\gamma - \lambda))\beta A\hat{k}^{\beta},$$

and a similar proposition to Prop. 1 is obtained. Therefore, for a range of parameter values, multiplicity of long-run equilibria arises because of the positive externality of environmental quality on consumer's welfare.

[12] The proof is straightforward and for this reason has been omitted from the paper.

Next, we focus on the properties of the different BGPs. From now we assume that A is greater than $\tilde{A}$ and we focus on long-run values $\hat{k}_1$ and $\hat{k}_2$ of case (i). Using the optimal policy function for x defined by (15) is immediate that the ratio of consumption to physical capital, $\hat{x}_1$, associated with $\hat{k}_1$ is higher than the ratio of consumption to physical capital, $\hat{x}_2$, associated with $\hat{k}_2$. Moreover, both ratios, $\hat{x}_1$ and $\hat{x}_2$, are positive according to (15) since both $\hat{k}_1$ and $\hat{k}_2$ are lower than $\alpha/(1-\alpha)$. See Prop. 1 and Fig. 1. In this case, the transversality conditions are fulfilled because it is easy to show that they are equivalent to have a long-run equilibrium with a positive ratio of consumption to capital and a positive value of capital per unit of effective labor.

On the other hand, the rate of growth will be positive depending on the value of A. If we look at the rate of growth given by (19), it is pretty clear that for an enough high value of A, the rate of growth is positive in interval $(\bar{k}_1, \bar{k}_2)$ defined by equation

$$(\alpha\gamma + (1-\alpha)\lambda)Ak^{\alpha} = \rho(\lambda + \gamma k), \tag{28}$$

see Fig. 2, where $k = \alpha\lambda/(1-\alpha)\gamma$ yields the maximum rate of growth that can be reached by the economy. Moreover, it is also pretty obvious that $\bar{k}_1$ decreases and $\bar{k}_2$ increases when A increases as an increase in A shifts the LHS of Eq. (28) upward and that $\lim_{A\to+\infty} \bar{k}_1 = 0$ and $\lim_{A\to+\infty} \bar{k}_2 = +\infty$.

$\Rightarrow$ *FIGURE* 2 $\Leftarrow$

Thus, to find out whether the rate of growth is positive we need to establish the relative position of $\bar{k}_1, \bar{k}_2, \hat{k}_1$ and $\hat{k}_2$ on the horizontal axis of Fig. 1 and 2. First, we focus on $\hat{k}_2$. This value increases with A since an increase in A shifts the RHS of Eq. (27) upward. For this reason, we have to expect that for an high enough value of A, $\hat{k}_2$ is higher than $\bar{k}_1$ that decreases with A. On the other hand, both $\bar{k}_2$ and $\hat{k}_2$ increase with A but whereas $\lim_{A\to+\infty} \bar{k}_2 = +\infty$, we have that $\lim_{A\to+\infty} \hat{k}_2 = \alpha/(1-\alpha)$ then we have to expect also that for an high enough value of A, $\hat{k}_2$ be lower than $\bar{k}_2$.

Thus, we have

Proposition 2 *For a high enough value of A, at least the rate of growth, $\hat{g}_2$, associated with $\hat{k}_2$, is positive.*

With a positive rate of growth, the environmental quality only can be preserved when $\gamma \geq \lambda$ according to function (4) and growth is only compatible with the EKC, that requires that environmental quality ultimately increases, when $\gamma > \lambda$. As we have assumed that γ is greater than λ, environmental quality is increasing along the BGP. As regards the sign of the rate of growth associated with $\hat{k}_1$ we cannot say nothing with generality except that in this case it cannot be avoided the possibility of a non-growing long-run equilibrium.[13]

Thus, we have two long-run equilibria, one with a relative high value of capital per unit of effective labor and a low ratio of consumption to capital and the other with a relative low value of capital per unit of effective labor and a high ratio of consumption to physical capital, and, we can state that, at least, the former supports a BGP with a positive rate of growth and decreasing pollution.

3.2 Stability of the long-run equilibrium

We will now investigate local stability around the BGPs. To analyze stability we rewrite Eq. (26) as follows

$$\dot{k} = \frac{(LHS(k) - RHS(k))p_2(k)k}{D(k)}, \tag{29}$$

where

$$\begin{aligned} LHS(k) &= \phi\rho(\lambda + (\lambda+\gamma)k + \gamma k^2), & (30) \\ RHS(k) &= (1+\phi(\gamma-\lambda))(\alpha - (1-\alpha)k)Ak^{\alpha}, & (31) \\ D(k) &= \phi(\lambda+\gamma k)((1+k)p_1(k) - p_2(k)), & (32) \end{aligned}$$

and $LHS(k)$ stands for the left-hand side of Eq. (27) and $RHS(k)$ for the right-hand side. Then, stability is given by the sign of

$$\begin{aligned} \frac{d\dot{k}}{dk} &= \frac{1}{D^2}\left(\left((LHS' - RHS')p_2k + (LHS - RHS)(p_2'k + p_2)\right)D \right. \\ &\quad \left. +(LHS - RHS)p_2kD'\right), \end{aligned}$$

[13]Eq. (19) could give a negative growth rate but then the non-negativity constraint applies. In fact, this is the result obtained in the numerical example.

evaluated at the stationary point $\hat{k}_2$ that yields

$$\frac{d\dot{k}}{dk} = \frac{(LHS'(\hat{k}_2) - RHS'(\hat{k}_2))p_2(\hat{k}_2)\hat{k}_2 D(\hat{k}_2)}{D(\hat{k}_2)^2} \tag{33}$$

since by definition $LHS(\hat{k}_2) - RHS(\hat{k}_2)$ is zero at the stationary point.

From Fig. 1, it is pretty clear that $LHS'(\hat{k}_2) > RHS'(\hat{k}_2)$. Moreover, this relationship holds even if $\hat{k}_2$ is on the left of $\alpha^2/(1-\alpha^2)$, the maximum of the $RHS(k)$ represented in Fig.1. Next, we study the sign of $p_2(k)$. As $p_2(k)$ is a quadratic, strictly convex function with a negative independent term, $p_2(k) = 0$ presents a unique positive root given by

$$\hat{k} = \frac{-((1-\alpha)\lambda - \alpha\gamma) + \sqrt{((1-\alpha)\lambda - \alpha\gamma)^2 + 4\gamma(1-\alpha)\alpha\lambda}}{2\gamma(1-\alpha)} = \frac{\alpha}{1-\alpha}, \tag{34}$$

so that $p_2(k)$ is negative on the left of $\alpha/(1-\alpha)$, the intersection point of the $RHS(k)$ with the horizontal axis, and hence $p_2(\hat{k}_2)$ is also negative since according to Prop. 1 $\hat{k}_2$ is lower than $\alpha/(1-\alpha)$. Thus, to determine the sign of (33), what we need is to find out the sign of $D(\hat{k}_2)$.

The sign of $D(\hat{k}_2)$ depends on the sing of $(1+k)p_1(k) - p_2(k)$ given that $\lambda + \gamma k$ is positive for $\hat{k}_2$ positive. See (32) above. Developing this expression we obtain the following three-degree polynomial in k

$$\begin{aligned}(1+k)p_1(k) - p_2(k) &= (1-\alpha)^2\gamma k^3 - \alpha((3-2\alpha)\gamma + (1-\alpha)\lambda)k^2 \\ &\quad -(1-\alpha)(\alpha\gamma + (1+2\alpha)\lambda)k + \alpha^2\lambda.\end{aligned}$$

The sign of this polynomial can be evaluated at $k = \alpha^2/(1-\alpha^2)$ and at $k = \alpha/(1-\alpha)$, yielding respectively

$$\begin{aligned}&\left(1+\frac{\alpha^2}{1-\alpha^2}\right)p_1\left(\frac{\alpha^2}{1-\alpha^2}\right) - p_2\left(\frac{\alpha^2}{1-\alpha^2}\right) \\ &= -\frac{(1-\alpha)\alpha^3}{(1-\alpha^2)^3}\left((1+\alpha^2)\gamma + (1-\alpha^2)\lambda\right) < 0,\end{aligned} \tag{35}$$

$$\left(1+\frac{\alpha}{1-\alpha}\right)p_1\left(\frac{\alpha}{1-\alpha}\right) - p_2\left(\frac{\alpha}{1-\alpha}\right) = -\frac{\alpha}{(1-\alpha)^2}(\alpha\gamma + (1-\alpha)\lambda) < 0. \tag{36}$$

Then, as these two values of the polynomial are negative and the independent term is positive, equation $(1+k)p_1(k) - p_2(k) = 0$, according to Descartes' rule of signs for polynomial equations, has two positive roots being the polynomial positive on the left of the lower root and on the right of the higher root. Thus, the polynomial must be negative in interval $[\alpha^2/(1-\alpha^2), \alpha/(1-\alpha)]$ and consequently $D(k)$ is negative, at least, in this interval and we can conclude that

Proposition 3 *If $\hat{k}_2$ belongs to interval $[\alpha^2/(1-\alpha^2), \alpha/(1-\alpha)]$, then the long-run equilibrium $(\hat{k}_2, \hat{x}_2, \hat{g}_2)$ is unstable.*

If $\hat{k}_2$ is on the left of $\alpha^2/(1-\alpha^2)$ surely it is also unstable and if it is enough close to $\alpha^2/(1-\alpha^2)$ on the left it is also unstable without doubt. As regards the other equilibrium $(\hat{k}_1, \hat{x}_1, \hat{g}_1)$ nothing can be said with generality although we expect that this equilibrium is also unstable. This conjecture is based on the fact that along the BGP our model behaves as an AK model since the output can be written as a linear function of capital, $Y = A\hat{k}^{\alpha-1}K$, and it is well-known that the lack of transitional dynamics characterizes this kind of growth model so that we expect that our model inherits this property for the different equilibria. Moreover, this is the case for the numerical example developed in Section 4.1.

3.3 The effects on growth of greener preferences

Next, we study the effects of a change in ϕ on the BGPs. In this Section we focus on the long-run equilibrium value $\hat{k}_2$ that, under some conditions, presents a positive rate of growth. In our model for a given combination (Q, C), ϕ determines the value that the individual places on one extra unit of environmental services in terms of consumption therefore a rise in ϕ can be interpreted as a change in consumer's preferences for a cleaner environment.

The analysis of the effects of greener preferences can be easily conducted from Eq. (27) if we divide both sides of the equation by ϕ. The result is that in this case only the RHS of the equation depends (negatively) on ϕ. Then as the two critical values of the RHS, the maximum and the intersection point, do not depend on ϕ, an increase in ϕ shifts the RHS curve downward causing a decrease in $\hat{k}_2$.

The effect on the ratio of consumption to capital can be evaluated also easily using optimal policy function (15). In this case we find that greener preferences have an ambiguous effect on $\hat{x}_2$. Finally, (19) can be used to evaluate the effect on the growth rate. As (19) does not depend on ϕ, the effect on the rate of growth depends on whether $\hat{k}_2$ is greater or lower than $\alpha\lambda/(1-\alpha)\gamma$, the value for k that yields the maximum rate of growth. See Fig. 2. Thus, although there exists the possibility that greener preferences have a positive effect on the growth rate of the economy, the sign of the effect is not determined and a change in consumer's preferences for a cleaner environment could also cause a negative effect on growth. Moreover, it seems clear that the greener the preferences, the lower the possibilities that a change in preferences has a positive effect on growth. As an increase in ϕ reduces $\hat{k}_2$ the higher ϕ, the lower $\hat{k}_2$, then according to Fig. 2, the higher the possibilities that $\hat{k}_2$ is lower than $\alpha\lambda/(1-\alpha)\gamma$. Thus, for a great enough value for ϕ, it should be expected that $\hat{k}_2$ is lower than $\alpha\lambda/(1-\alpha)\gamma$ and in that case any change in the preferences for a cleaner environment would cause a reduction on the growth rate. Similar results were obtained by Smulders and Gradus (1996) in their model with pollution abatement.

4 The Environmental Kuznets Curve

As along the BGP the marginal productivity of capital and the ratio of output to capital must be constant we can represent the BGP using the production function as it shown in Fig. 3.

$$\Rightarrow FIGURE\ 3 \Leftarrow$$

The figure shows how the technological progress moves up the curve of production and how the BGP line goes through all the points with the same productivity of capital. From the figure is clear the dynamics of capital depends on its initial value. If K_0 is lower than $\hat{K}_0$, that it is defined as $\hat{K}_0 = \hat{k}z_0$, then there is transitional dynamics to the BGP. Notice that investment in R&D is irreversible so that it is not possible to moves down the production function. Thus, in the figure the first production curve is a technological frontier defined by the dirtiest technology available.

Thus, if K_0 is lower than $\hat{K}_0$ as it occurs in Figure 3, the shadow price of a cleaner technology is low relative to the cost of developing it and as we pointed out in Section 3, the technological development is not worth investing in. Then during a first stage, $I_z = 0$, and the economy evolves according to the following differential equations

$$\frac{\dot{\nu}}{\nu} = \rho - \frac{U_Q}{U_C}Q_K - F_K, \quad \frac{\dot{K}}{K} = F(z_0, K) - \frac{C}{K},$$

that characterize the optimal capital accumulation without technological progress in a polluted environment.[14]

For the functions of the model, these two conditions yield:

$$\frac{\dot{C}}{C} = \alpha A \left(\frac{z_0}{K}\right)^{1-\alpha} - \lambda\phi\frac{C}{K} - \rho, \tag{37}$$

$$\frac{\dot{K}}{K} = A \left(\frac{z_0}{K}\right)^{1-\alpha} - \frac{C}{K}, \tag{38}$$

which describe the transitional dynamics of the economy. Without population growth or exogenous technological progress, this model does not present a BGP. For this reason, we conduct the analysis of the transitional dynamics in this Section in terms of the levels of C and K.

The dynamic system has a unique positive steady state given by

$$K^* = \left(\frac{A(\alpha - \lambda\phi)}{\rho}\right)^{\frac{1}{1-\alpha}} z_0, \quad C^* = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha - \lambda\phi}{\rho}\right)^{\frac{\alpha}{1-\alpha}} z_0, \tag{39}$$

provided that $\alpha > \lambda\phi$.

To analyze the stability of the dynamic system we rewrite it as follows

$$\dot{C} = \alpha A \left(\frac{z_0}{K}\right)^{1-\alpha} C - \lambda\phi\frac{C^2}{K} - \rho C, \tag{40}$$

$$\dot{K} = A K^{\alpha} z_0^{1-\alpha} - C, \tag{41}$$

[14] See for instance the papers by Forster (1973) or Selden and Song (1995).

and calculate their partial derivatives:

$$\frac{\partial \dot{C}}{\partial C} = \alpha A\left(\frac{z_0}{K}\right)^{1-\alpha} - 2\lambda\phi\frac{C}{K} - \rho, \tag{42}$$

$$\frac{\partial \dot{C}}{\partial K} = \frac{C}{K}\left(\alpha(\alpha-1)A\left(\frac{z_0}{K}\right)^{1-\alpha} + \lambda\phi\frac{C}{K}\right), \tag{43}$$

$$\frac{\partial \dot{K}}{\partial C} = -1 < 0, \tag{44}$$

$$\frac{\partial \dot{K}}{\partial K} = \alpha A\left(\frac{z_0}{K}\right)^{1-\alpha} > 0. \tag{45}$$

Partial derivatives (42) and (43) can be simplified using stationary condition, $\dot{C} = 0$. From this condition we know that $\lambda\phi C/K = \alpha A(z_0/K)^{1-\alpha} - \rho$, that allows us to rewrite (42) and (43) as follows

$$\frac{\partial \dot{C}}{\partial C} = -\lambda\phi\frac{C}{K} < 0, \quad \frac{\partial \dot{C}}{\partial K} = \frac{C}{K}\left(\alpha^2 A\left(\frac{z_0}{K}\right)^{1-\alpha} - \rho\right).$$

Then, if we evaluated these derivatives at the steady state taking into account that

$$\frac{z_0}{K^*} = \left(\frac{\rho}{A(\alpha-\lambda\phi)}\right)^{\frac{1}{1-\alpha}}, \quad \frac{C^*}{K^*} = \frac{\rho}{\alpha-\lambda\phi},$$

we obtain the following expression for the Jacobian determinant: $|J_{SS}| = -(1-\alpha)\rho^2/(\alpha-\lambda\phi)$, that is negative. This implies that the two roots of the characteristic equation have opposite signs, which establishes the steady state is locally a saddle point. Thus, we obtain

Proposition 4 *If $\alpha > \lambda\phi$ the dynamic system (40)-(41) has a unique positive steady state that is saddle point.*

For this kind of stationary points there are two stable branches leading to the steady state in the phase diagram, and then there exists an optimal path to approach the steady state as it is illustrated in Fig. 4.

$\Rightarrow$ *FIGURE* 4 $\Leftarrow$

However, the stable path is not so relevant in our analysis as it is the localization of the transition point to the BGP defined by the interior solution and the localization of the initial stock of capital. As the figure shows the transition path must not be necessarily on the stable path. The transition point is defined by the following values for the capital and consumption: $\hat{K}_0 = \hat{k}z_0$ and $\hat{C}_0 = \hat{x}\hat{K}_0$. Then if the transition point belongs to the area of the phase diagram where both the capital and consumption are increasing, there will exist a unique path that leads to the transition point provided that $K_0 < \hat{K}_0$. Thus, there is a first stage during which the capital, consumption and pollution rise monotonically and the economy reaches the transition point $(\hat{K}_0, \hat{C}_0)$ in finite time. This path in Fig. 4 corresponds to the movement along the production curve $F(z_0, K)$ from K_0 in Fig. 3. Once, the transition point is reached, the economy grows at the rate of technological progress and the environmental quality increases. The result is that during a first stage, the environmental quality is decreasing to increase later yielding an EKC.

Finally, we would like to clarify that an EKC can appear even if condition $\alpha > \lambda\phi$ of Prop. 4 is not satisfied since the only condition we need to obtain a first stage with increasing pollution is that the transition point is in the area of the phase diagram where both the capital and consumption are increasing.

4.1 Numerical example

We finish this Section with a numerical illustration whose aim is to show the possibility of our theoretical results, rather than to account for reality. Table 1 summarizes the parameters upon which our simulations are based.

$$\Rightarrow \ TABLE\ 1 \ \Leftarrow$$

Following Cassou and Hamilton (2004) we assume that the negative effect of pollution on production reduces β by 0.05 that for a standard value of 0.4 for β yields $\alpha = 0.35$. The remaining parameters that describes preferences and technology in the baseline calibration are selected simultaneously to satisfy condition $\beta + \chi(\gamma - \lambda) = 1$, to match a growth rate of 0.02, a ratio of consumption to capital in the interval $[0.15, 0.25]$ and a growth rate of the environmental quality along the BGP in the interval $[0.02, 0.06]$. For the values of Table 1 the values that characterize the BGP are $\hat{k} = 0.51$ and $\hat{x} = 0.16$.[15]

[15] The range of the interval for the ratio of consumption to capital includes acceptable values for this

Observe that $\hat{k}$ belongs to interval $[\alpha^2/(1-\alpha^2) = 0.1396, \alpha/(1-\alpha) = 0.5385]$ so that according to Prop. 3 the long-run equilibrium $(\hat{k} = 0.51,\ \hat{x} = 0.16,\ \hat{g} = 0.02)$ is unstable.

If we normalize $z_0 = 1$, $\hat{K}_0$ in Fig. 3 is equal to $\hat{k} = 0.51$, then if the initial value of capital is lower than this figure, the economy will evolve during a first stage according to the first-order differential equations system given by (40)-(41):

$$\dot{C} = 4.9525 \times 10^{-2} \frac{C}{K^{0.65}} - 3.325 \times 10^{-3} \frac{C^2}{K} - 0.0562C, \tag{46}$$

$$\dot{K} = 0.1415K^{0.35} - C. \tag{47}$$

We have solved this system by the time elimination method using Matlab 7.0 for $K_0 = 10^{-4}$ and the transition values $\hat{K}_0 = 0.51$ and $\hat{C}_0 = \hat{x}\hat{K}_0 = 0.0816$. Notice that, as we do not want to approach the steady state represented in Fig. 4 but the transition point, it is not necessary to use the eigenvector procedure to determine the slopes of the policy function, $C(K)$, at the steady state. See Mulligan and Sala-i-Martín (1993) for more details about this procedure. The policy function obtained applying this method is showed in Fig. 5 and corresponds to the path drawn in Fig. 4 that leaves from the initial stock of capital. This means that the transition point $(\hat{K}_0, \hat{C}_0)$ is in the area of the phase diagram where both the capital and consumption are increasing and that, moreover, it can be reached from the initial value of capital.

$\Rightarrow$ *FIGURE* 5 $\Leftarrow$

Once the policy function $C(K)$ has been computed, time must be reintroduced. The time path of K can be computed by solving the initial-value problem

$$\dot{K} = 0.1415K^{0.35} - C(K), \quad K(0) = 0.0001. \tag{48}$$

Then, we compute the time T at which $K(T) = \hat{K}_0$. Thus, during the interval $[0, T]$, $k(t)$ is equal to $K(t)$ since we have normalized $z_0 = 1$. From time $t = T$ on, the optimal path is $k(t) = \hat{k} = 0.51$.

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variable in the developed economies. On the other hand, the range of the interval for the rate of growth of the environmental quality could include lower values both for the upper and lower limits without causing any qualitative change in the dynamics of the environmental quality that is showed in Fig. 6.

Finally, environmental services are given by function $Q(t) = K(t)^{-0.25}$ during the interval $[0, T]$ so that environmental quality decreases with capital until its transition value, $\hat{Q} = \hat{K}_0^{-0.25}$, is reached at T. From time $t = T$ on, environmental quality is increasing because of the investment in abatement technologies. The result is an Environmental Kuznets Curve as the one showed in Fig. 6.

$\Rightarrow$ *FIGURE* 6 $\Leftarrow$

Next, we conclude this section developing a sensitivity analysis of the results. Firstly, we would like to point out that for the benchmark parameters values a change in consumer preferences for a cleaner environment has a positive effect on growth. For these values, the value of k that maximizes the growth rate in Fig.2 is equal to 0.04, clearly below the steady-state value for k. In this case, a marginal increase in ϕ reduces k causing a raise in the growth rate. Moreover, it is easy to check that this will occur for any ϕ greater than the benchmark value. Using (27) we find that $lim_{\phi \to +\infty}\ k = 0.17643$ that is lower than the value of k that maximizes the growth rate, thus for any $\phi \in [0.0133, +\infty)$, $\hat{k} \in (\alpha\lambda/(1-\alpha)\gamma, \bar{k}_2)$ in Fig. 2 and an increase in ϕ raises the growth rate of the economy. Thus, we can conclude that $\hat{k}$ is decreasing with respect to ϕ and $\hat{g}$ is increasing with respect to ϕ in the interval $[0.0133, +\infty)$. To go further in the sensitivity analysis we have solved the model for the combination of the following values of the parameters $\phi = (0.0133,\ 0.0633,\ 0.1133,\ 0.1633,\ 0.2133)$, $\alpha = (0.325,\ 0.350,\ 0.375)$ and $\rho = (0.0400,\ 0.0500,\ 0.0562)$. For the forty-five cases analyzed, we have found the following regularities. A change in consumer preferences for a cleaner environment has a negative effect on the ratio of consumption to capital whereas an increase in the rate of discount has a positive effect on this ratio but a negative effect on the capital per unit of effective labor and on the rate of growth. These are standard effects for a variation in the rate of discount. Finally, we have considered different values for α in order to take into account different impacts of the environment on production. The results of the sensitivity analysis indicate that the greater the impacts of the environment on production, i.e. the lower is α, the greater the growth rate and the ratio of consumption to capital and the lower the capital per unit of effective labor. Thus, we have that when the environmental quality enhances the productivity of capital and labor, the greater the effects of the environmental quality on production the greater the effects of investment in R&D on productivity which allows to support greater rates of growth for the economy with greater ratios of consumption to capital based in cleaner technologies. Finally, we would like to highlight that for all the cases our algorithm works provided the initial stock of capital is low and the optimal investment pattern yields an U-shaped profile for the dynamics of the environmental quality. Thus, the EKC can be seen as a natural corner solution for an economy that evolves from a pristine natural environment.

5 Conlusions

This paper has investigated socially optimal patterns of economic growth and pollution in a neoclassical growth model with endogenous technological progress. In the model we assume that quality of natural environment is (indirectly) a production factor. Moreover, we assume that investment in environmentally oriented R&D allows to produce the output with a cleaner technology but, as pollution affects negatively production, the investment in R&D also enhances the productivity of capital and labor. In this framework, if the initial stock of capital is low, and consequently consumption is also low, it is not profitable to invest in R&D and all the investment effort is directed to accumulate capital and increase consumption resulting in an increase of pollution. This increases the shadow price of a cleaner technology and reduces the cost of developing it until that it is profitable to invest in R&D and use a cleaner technology. Then if the new technologies are efficient to compensate the negative effects on the environment of the accumulation of capital, it is possible to growth and increase environmental quality. The result is that the optimal investment pattern in capital and R&D supports an inverted-U-shaped pattern of pollution to countries' income. The second part of this pattern is explained because our model behaves an AK model along the BGP so that the marginal productivity of capital becomes constant. We also find that the effect of a change in consumer's preferences for a cleaner environment has an effect on the growth rate that depends on the degree of environmental conscience of consumers. However, for the numerical example this condition does not work and we obtain that there is no conflict between the environmental preservation and the economic growth. The reason that explains the positive influence of greener preferences on growth is that greener preferences increase the investment in R&D causing an increase in productivity that can support a greater rate of growth for the economy. In the numerical example this occurs for any the degree of consumers' environmental conscience .

The main lesson for the design of the climate change policies that can be learnt for our model is that we cannot trust only on economic growth to face this problem. Polluting firms can be interested in investment in abatement technologies because of the positive effects that the environmental quality has on production. However, even in this case, the investment is not going to be optimal since the environmental quality is a public good. In fact, for the case of the climate change, the environmental quality is a global public good and we have to expect that the decentralized equilibrium is pretty far from the optimal allocation of the resources to the R&D sector of the economy. Thus the environmental policy should close this gap promoting energetically the investment of the polluting firms in abatement technologies.

Different lines for future research can be envisaged. First, we would like to consider a more general utility function that admits different values for the elasticity of the intertemporal substitution. Nevertheless, we guess that this is not a key parameter in the explanation of the EKC presented in this paper that is based on technological change. Another obvious extension is to study how the social planner solution can be implemented through different instrument of environmental policy. Another direction that could be developed is to consider a bisectorial growth model that allows us to incorporate different types of technological progress in the line of the papers published by Cassou and Hamilton (2004), Hart (2004), Grimaud and Rouge (2008) and Reis et al. (2008). However, as Cassou and Hamilton (2004) have shown, we think that the same profile for the dynamics of the environmental quality will be obtained but in this case explained by a change in the sectorial composition of the total output. Finally, it could be also considered that environmental damages are related to a stock pollutant.

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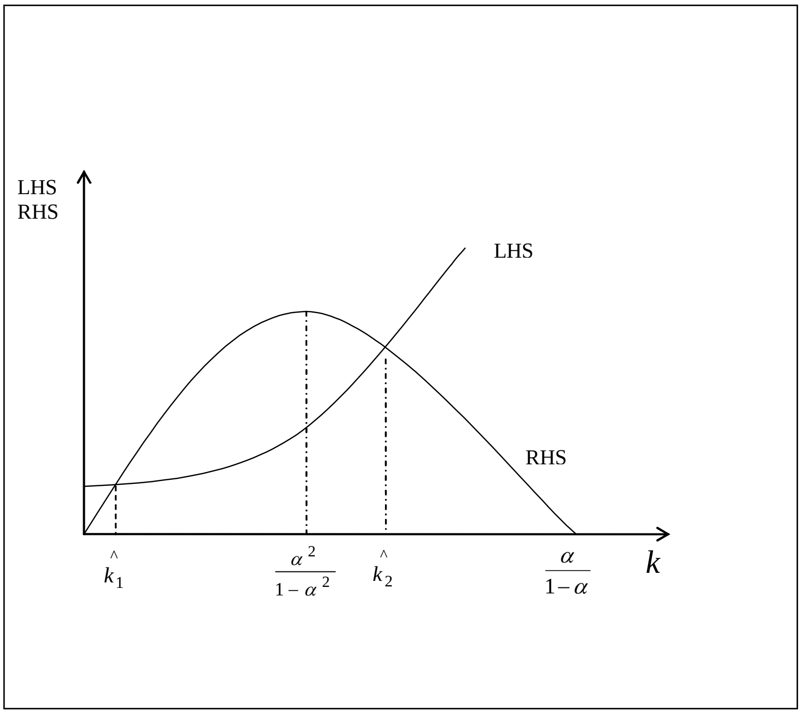

Figure 1. Steady-state values for k.

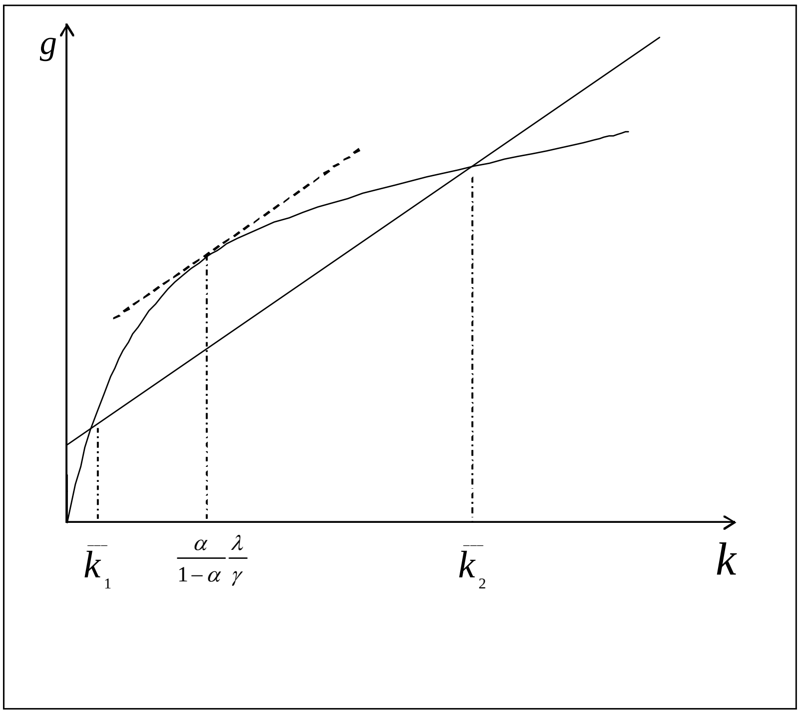

Figure 2. Interval for k with positive growth rates.

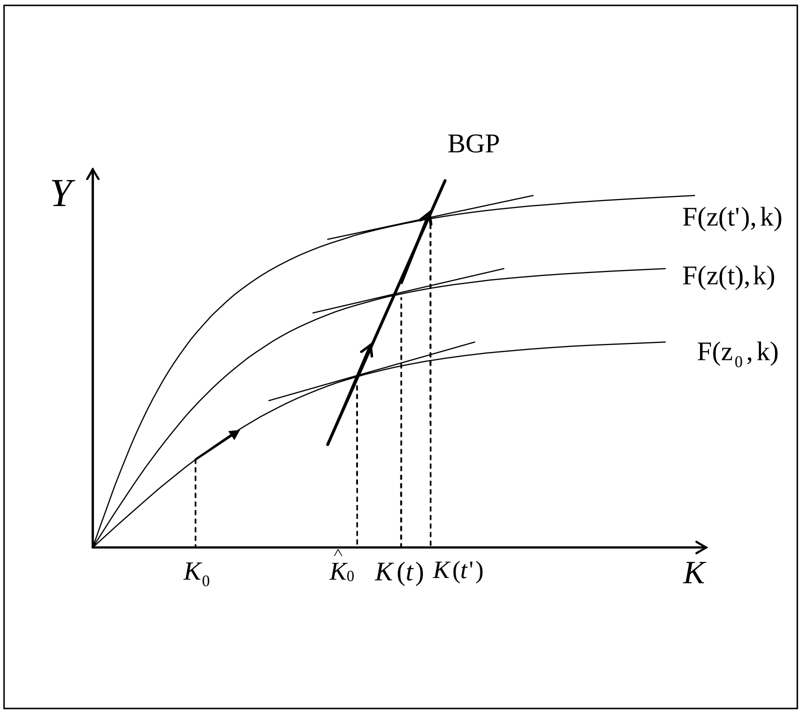

Figure 3. The BGP.

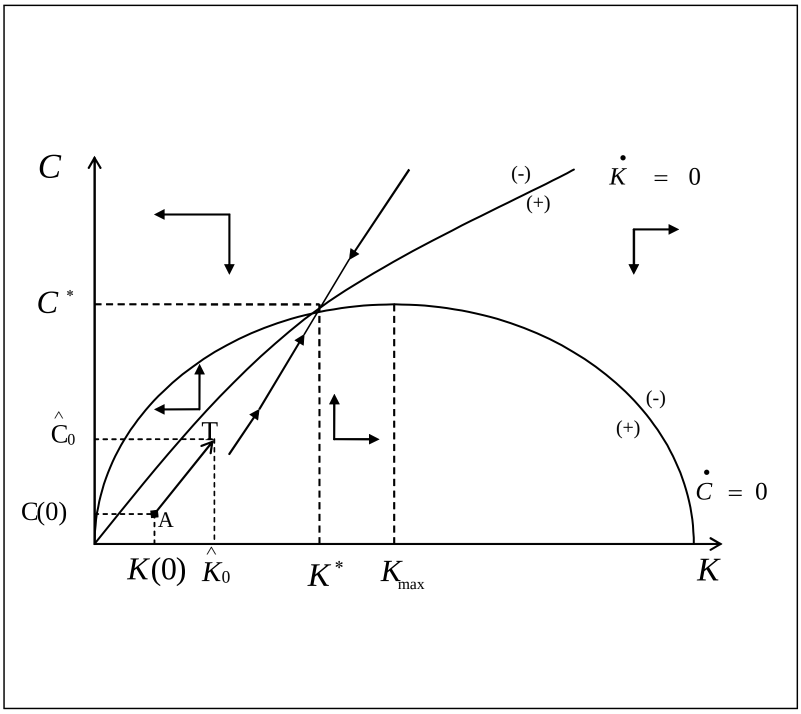

Figure 4. Phase diagram in (K, C) space. Case $\alpha > \lambda\phi$.

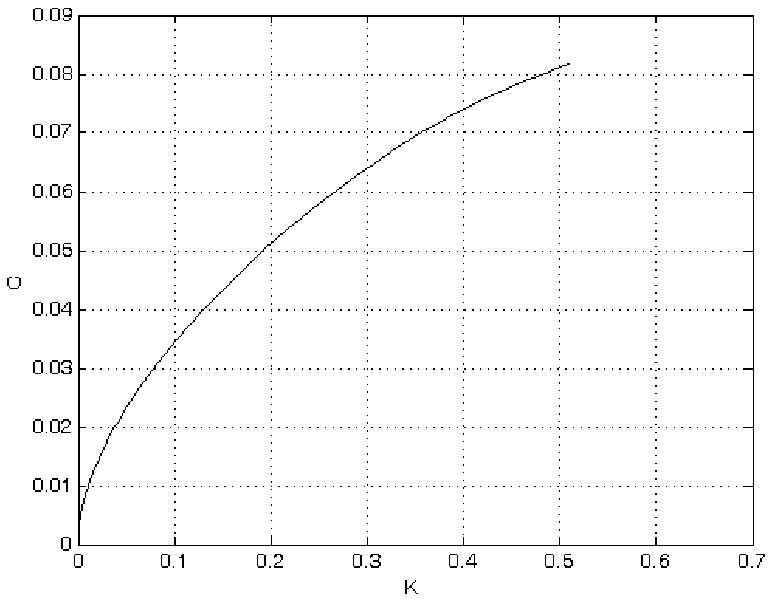

Figure 5. Policy function $C(K)$

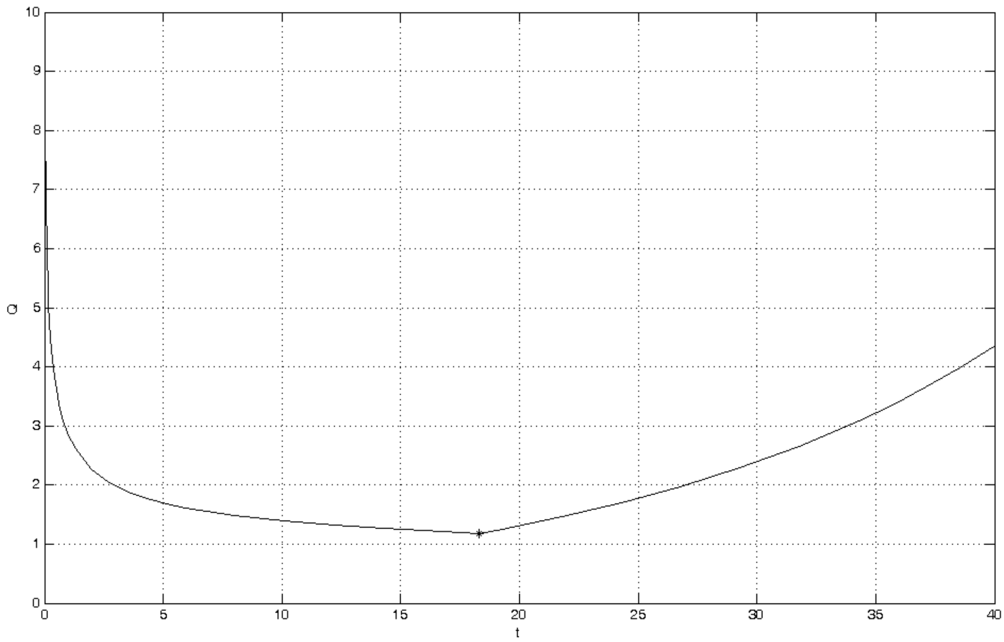

Figure 6. The Environmental Kuznets Curve

Preference parameters	$\phi = 0.0133$	$\rho = 0.0562$		
Technological parameters	$A = 0.1415$	$\alpha = 0.3500$	$\lambda = 0.2500$	$\gamma = 3.2500$

TABLE 1. Benchmark parameters